Otto von Guericke Universität Magdeburg **Winter Term 2018**
Fakultät II – Mathematik und Naturwissenschaften
Institut für Mathematik
Jan Heiland as of: October 26, 2018

Differential Algebraic Equations

Exercise Sheet 2 – Linear DAEs with constant coefficients

A Regularity and Kronecker Forms
Check whether the matrix pairs

$$\left(\begin{bmatrix}1 & 1 & 0\\ 0 & -1 & 1\\ 0 & 0 & 0\end{bmatrix}, \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & -1\\ 0 & -1 & 1\end{bmatrix}\right) \quad \text{and} \quad \left(\begin{bmatrix}2 & -1 & 1\\ 3 & -2 & 2\\ 0 & 0 & 0\end{bmatrix}, \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & -1\\ 1 & -1 & 1\end{bmatrix}\right)$$

are regular or singular and determine their Kronecker canonical forms by elementary row and column transforms.

B Index-1 condition
Show that the matrix pair

$$(E, A) = \left(\begin{bmatrix}I_r & 0\\ 0 & 0\end{bmatrix}, \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\right)$$

with E, $A \in \mathbb{C}^{n,n}$, and $r < n$, is of index 1 if, and only if, A_{22} is square and nonsingular.

C Regularity and commutativity
Let E, $A \in \mathbb{C}^{n,n}$ satisfy $EA = AE$. Show

1. that (E, A) is regular if, and only if, $\operatorname{kernel} E \cap \operatorname{kernel} A = \{0\}$
2. and that $\operatorname{ind}(E, A) = \operatorname{ind} E$.

D Regularity and commutativity II
Let (E, A) be regular with E, $A \in \mathbb{C}^{n,n}$. For a $\tilde\lambda$ such that $\tilde\lambda E - A$ is invertible, show

1. that $\tilde E := (\tilde\lambda E - A)^{-1}E$ and $\tilde A := (\tilde\lambda E - A)^{-1}A$ commute
2. and that $\operatorname{ind}(E, A) = \operatorname{ind} \tilde E$.

E Drazin inverse as group inverse
If $\operatorname{ind} E \leq 1$, then the Drazin inverse E^D is also called group inverse of E and denoted by $E^{\#}$. Show that $E \in \mathbb{C}^{n,n}$ is an element of *a* group $\mathbb{G} \subset \mathbb{C}^{n,n}$ with the matrix multiplication if and only if $\operatorname{ind} E \leq 1$, and that the inverse in such a group is just $E^{\#}$.

F Drazin inverse property
Prove that $((E^D)^D)^D = E^D$ for all $E \in \mathbb{C}^{n,n}$

G Equivalence and Regularity of Matrix Pairs
Show that

1. the relation of *strong equivalence* as defined in Definition 3.1 is an equivalence relation,
2. regularity of a matrix pair (Definition 3.5) is invariant under *strong equivalence*.

H Singular Blocks in the Kronecker Canonical Form
Explain how the singular blocks

$$\lambda \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} - \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

and

$$\lambda \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} - \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

fit into the *Kronecker Canonical Form* (Theorem 3.3).