Otto von Guericke Universität Magdeburg **Summer Term 2021**
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Differential Algebraic Equations

Exercise Sheet 2 – Linear DAEs with constant coefficients ctd.

A Index-1 condition
Show that the matrix pair

$$(E, A) = \left(\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \right)$$

with E, $A \in \mathbb{C}^{n,n}$, and $r < n$, is of index 1 if, and only if, A_{22} is square and nonsingular.

B Regularity and commutativity
Let E, $A \in \mathbb{C}^{n,n}$ satisfy $EA = AE$. Show

1. that (E, A) is regular if, and only if, $\operatorname{kernel} E \cap \operatorname{kernel} A = \{0\}$
2. and that $\operatorname{ind}(E, A) = \operatorname{ind} E$.

C Regularity and commutativity II
Let (E, A) be regular with E, $A \in \mathbb{C}^{n,n}$. For a $\tilde{\lambda}$ such that $\tilde{\lambda}E - A$ is invertible, show

1. that $\tilde{E} := (\tilde{\lambda}E - A)^{-1}E$ and $\tilde{A} := (\tilde{\lambda}E - A)^{-1}A$ commute
2. and that $\operatorname{ind}(E, A) = \operatorname{ind} \tilde{E}$.

D Drazin inverse as group inverse
If $\operatorname{ind} E \leq 1$, then the Drazin inverse E^D is also called group inverse of E and denoted by $E^{\#}$. Show that $E \in \mathbb{C}^{n,n}$ is an element of *a* group $\mathbb{G} \subset \mathbb{C}^{n,n}$ with the matrix multiplication if and only if $\operatorname{ind} E \leq 1$, and that the inverse in such a group is just $E^{\#}$.

D Drazin inverse property
Prove that $((E^D)^D)^D = E^D$ for all $E \in \mathbb{C}^{n,n}$