as of: May 18, 2021

# Differential Algebraic Equations Exercise Sheet 3 – Linear DAEs with Time-varying Coefficients

## A (Local) Equivalence Transformation

Prove that the (local) equivalence transformation defined in Definition 4.6 in the lecture defines an equivalence relation.

B (Global) Equivalence and Solvability

Compute  $\vec{E} = PEQ$  and  $\vec{A} = PAQ - PEQ$  for

$$E(t) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A(t) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad P(t) = \begin{bmatrix} t & 1 \\ 1 & 0 \end{bmatrix}, \quad Q(t) = \begin{bmatrix} -1 & t \\ 0 & -1 \end{bmatrix},$$

and for

$$E(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad A(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad P(t) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad Q(t) = \begin{bmatrix} 0 & -1 \\ 1 & -t \end{bmatrix},$$

compare it to the initial examples of Section 4 of the lecture, and interpret your observations.

### C Characteristic Quantities I

Determine the (local) characteristic quantities (r, a, s) of

$$(E(t), A(t)) = \left( \begin{bmatrix} 0 & 0 \\ 1 & \eta t \end{bmatrix}, \begin{bmatrix} -1 & -\eta t \\ 0 & -(1+\eta) \end{bmatrix} \right)$$
(1)

for every  $t \in \mathbb{R}$  and for every  $\eta \in \mathbb{R}$ .

## **D** Characteristic Quantities II

Determine the (local) characteristic quantities (r, a, s) of

$$(E(t), A(t)) = \left( \begin{bmatrix} 0 & 0 \\ 0 & t \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$
(2)

for every  $t \in \mathbb{R}$ .

#### E Drazin Inverse

For  $E \in \mathcal{C}(I, \mathbb{R}^{n,n})$ , we define the Drazin inverse pointwise  $E^D$  pointwise via  $E^D(t) = E(t)^D$ . Determine the  $E^D$  for the matrix functions E from (1) and (2). What do you observe?

# F Global Characteristic Quantities

Compute the (global) characteristic quantities  $(r_i, a_i, s_i)$ ,  $i = 1, ..., \mu$ , of the pair of matrix functions (E, A) given in (1).